The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

Write the expressions in problems 1 and 2 as the quotient of two relatively prime integers.

1. \( \frac{33}{40} \) = 

2. \( \frac{1 + \frac{3}{6}}{\frac{4}{9} + \frac{2}{3}} = \frac{17,784}{11} \)

3. Carrie A. Handful found on line 14 of her 2003 Minnesota Income Tax that her taxable income was $243,812. She then read, "If line 14 is over $110,390, enter on line 15 $7310.24 + 7.85\% of the amount over $110,390." How much should she enter on line 15?

4. The set of three positive integers \( \{12, 45, m\} \) has a greatest common divisor \( d > 1 \) and a least common multiple of 2520. What is the sum of the possible values for the integer \( m \)?

1. Let \( x = .272727 \ldots \) 
   \[
   \begin{align*}
   100x &= 27.272727\ldots = 27 + x \\
   99x &= 27 \\
   x &= \frac{27}{99} = \frac{3}{11}
   \end{align*}
   \]

2. \[
   \begin{align*}
   \left( \frac{1 + \frac{3}{2}}{\frac{4}{3} + \frac{2}{3}} \right) &= \frac{8}{10} = \frac{4}{5} \\
   \left( \frac{3}{2} \right)^2 &= \frac{9}{4} \\
   &= \frac{33}{40}
   \end{align*}
   \]

3. \( \frac{243,812}{110,390} = \frac{2 \cdot 3 \cdot 3 \cdot 5}{7 \cdot 310.24} \) 

4. \( \{2 \cdot 2 \cdot 3, 3 \cdot 3 \cdot 5, m\} \) 
   Since the \( \gcd > 1 \), it must be 3. Since \( \text{lcm} = 2^3 \cdot 3^2 \cdot 5 \cdot 7 \), \( m \) must have factors of \( 3, 2^3 \), and 7. It might have as additional factors 3 and 5. Possible:
   \[
   \begin{align*}
   168 &= 168 \\
   168 \cdot 3 &= 504 \\
   168 \cdot 5 &= 840 \\
   168 \cdot 3 \cdot 5 &= 2520 \\
   \frac{4032}{11}
   \end{align*}
   \]
Solutions

Minnesota State High School Mathematics League
Individual Event

2004-05 Event 1B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. Parallel lines pass through the vertices A and C of square ABCD as shown in Figure 1. If \( \angle ECD = 64^\circ \), what is the measure of \( \angle FAD? \)

2. Isosceles \( \triangle ABC \) has as its apex \( \angle A = 90^\circ \). BD is drawn so that \( \angle CB = \frac{1}{3} \angle CBA. \)
Line CE is drawn perpendicular to AC and meets the extension of BD at E (Figure 2). What is the measure of \( \angle BEC? \)

3. In \( \triangle ABC \) with \( AC < BC \), let the measure of the angles at A and B be \( \alpha \) and \( \beta \) respectively. Extend CA to D so that \( CD = CB \) (Figure 3). Express the measure of \( \angle CDB \) in terms of \( \alpha \) and \( \beta \).

4. In isosceles \( \triangle ABC \) with base BC and D a point on leg AC, \( BC = BD = DA \). What is the measure of \( \angle BAC? \)

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![Math Diagram](image.png)

Let \( \alpha = \angle BCD \)
\( \triangle BCD \) isosceles \( \Rightarrow \angle CBD = 180 - 2\alpha \)
\( \triangle ABC \) isosceles \( \Rightarrow \angle A = 180 - 2\alpha \)
\( \triangle BDA \) isosceles \( \Rightarrow \angle ABD = 180 - 2\alpha \)
\( \alpha = \angle ABD + \angle CBD = \frac{180 - \angle C}{2} (180 - 2\alpha) \)
\( 5\alpha = 360 \) \( \Rightarrow \alpha = 72 \) \( \Rightarrow \angle A = 180 - 144 = 36 \)
Minnesota State High School Mathematics League
Individual Event

2004-05 Event 1C

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. Express in exact, simplified form the value of \( \cos \frac{5\pi}{6} + \tan \frac{\pi}{3} \).

2. The line from the origin \( O \) to the point \( A(-4,-2) \) forms with the positive \( x \)-axis an angle \( \alpha \) that is between \( \pi \) and \( \frac{3\pi}{2} \). If \( \csc \theta = \sqrt{5}, 0 < \theta < \frac{\pi}{2} \), express \( \alpha \) as a radian expression involving \( \theta \).

3. Two tangents to a circle, each of length 12, intersect to form an angle of 60°. If the tangents meet the circle at points \( A \) and \( B \), the length of the major arc \( AB \) is \( k\pi \). Give the exact, rationalized value of \( k \).

4. Hipparchus (190-120 B.C.) estimated the distance from the surface of the earth to the moon by the following method, described here using miles as the unit of measurement. Using an eclipse of the moon as the signal for obtaining simultaneous observations, it was found that when observers were separated by 6220 miles (measure in the usual way along the surface of the earth), observer \( A \) could see the moon \( M \) just over the horizon while observer \( B \) saw it directly overhead (Figure 1). Estimates of the radius of the earth at the time were 4000 miles. Using these numbers, what is \( BM \) to the nearest mile?
Minnesota State High School Mathematics League
Individual Event

2004-05 Event 1D

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

No calculators on this event.

1. What is the product of the roots of \(2x^2 - 3x + 2 = 0\)?

2. Find \(b\) so that the equation \(4x^2 + bx + 9 = 0\) has one rational (double) root.

(Contributed at the 2003 coaches conference)

Give full credit for \(b = 12\) or for \(b = -12\)

3. Given that \(-2\) is one root of \(6x^3 - 5x^2 - 29x + 10 = 0\), find the other two roots

4. The polynomial \(3x^3 - 13x^2 + ax + b = 0\), \(a\) and \(b\) real numbers, has \(2 - i\) as one of its roots. Where does the graph of \(y = 3x^3 - 13x^2 + ax + b\) cross the \(x\)-axis?

1. The product of the roots of \(4x^2 + bx + c = 0\) is \(c\). The product of the roots of \(x^2 - \frac{3}{2}x + 1 = 0\) is 1.

2. The roots are equal when the discriminant is 0.
\(b^2 - 4(4)(9) = 0\); \(b = \pm 12\)

3. Divide by \(x + 2\), synthetically or otherwise to get
\(6x^3 - 5x^2 - 29x + 10 = (x + 2)(6x^2 - 17x + 5)\)

Then \(6x^2 - 17x + 5 = (3x - 1)(2x - 5)\)

4. The sum of the roots of \(x^3 - \frac{13}{3}x^2 + \frac{13}{3}x + \frac{13}{3} = 0\) is \(-\frac{13}{3}\). Let the roots be \(2 - i\), \(2 + i\), and \(r\).
\((2 - i) + (2 + i) + r = \frac{13}{3}\)
\(r = \frac{13}{3} - 4 = \frac{1}{3}\)
Minnesota State High School Mathematics League
Team Event

2004-05 Meet 1

Each question is worth 4 points. Team members may cooperate in any way, but at the end of twenty minutes, one set of answers is to be submitted. Put answers on the lines provided.

1. Isosceles $\triangle ABC$ has as its apex $\angle A = 90^\circ$. $BD$ is drawn so that $\angle CBD = \frac{1}{3} \angle CBA$, and it is extended to meet a line erected at $C$, perpendicular to $AC$ at $E$ (Figure 1). If the legs of the triangle have length 1, what is the length of $CE$?

2. Isosceles $\triangle ABC$ has as its apex $\angle A = 90^\circ$. Adjacent trisectors of its angles meet to form $\triangle DEF$ as shown in Figure 2. What (in degrees) is the measure of $\angle DEC$?

3. For what value(s) of $x$ is $f(x) = |x - 3| + |1 - x|$ a minimum?

4. An observer on a boat $B$ traveling due east notices a light $20^\circ$ to the northeast. After traveling 1 mile, the light is $40^\circ$ to the northeast (Figure 4). Find the distance $x$ in miles that the boat will have to traveled when the light $L$ is due north.

5. The equation $9x^2 + 12x = 32$ can be written in the form $(x + b)^2 = k$. Find (i) $b$; (ii) $k$.

6. Give a numeric value for the continued fraction \[
\frac{3}{2 + \frac{3}{2 + \frac{3}{2 + \ldots}}}
\]
1. Label the angles as they were determined in Prob. 2 of Event B. Then
   \[ CD = 1 - AD = 1 - \frac{1}{\sqrt{3}} \]
   \[ CE = \sqrt{3} CD = \sqrt{3} - 1 \]

2. See Figure 2 on the Answer Sheet.
   \[ \angle DEC + 135^\circ + 75 + 60 = 360^\circ \]
   \[ \angle DEC = 90^\circ \]

4. See Figure 4 on the Answer Sheet.
   \[ \tan 20^\circ = \frac{h}{x+1} \quad \text{so} \quad h = x \tan 20 = \tan 20 \]
   \[ \tan 40^\circ = \frac{h}{x} \quad \text{so} \quad h = x \tan 40 = 0 \]
   \[ x \tan 40 - x \tan 20 = \tan 20 \]
   \[ x = \frac{\tan 20}{\tan 40 - \tan 20} = 0.766 \]

3. \((-\infty, -3]\]
   \[ \{ x \leq -3 \Rightarrow -x > 0 \} \quad f(x) = (-3 - x) + (1 - x) = -2 - 2x \]
   \[ [-3, 1] \]
   \[ \{ x > -3 \Rightarrow -x < 0 \} \quad f(x) = -(-3 - x) + (1 - x) = 4 \]
   \[ [1, \infty) \]
   \[ \{ x > -3 \Rightarrow -x < 0 \} \quad f(x) = -(3 - x) - (1 - x) = 2 + 2x \]

   \[ f \text{ achieves its minimum of } 4 \]
   \[ \text{for } x \text{ in } [-3, 1] ; \text{ i.e. } -3 \leq x \leq 1. \]

5. \[ q \left( x^2 + \frac{12}{9} x + \frac{4}{4} \right) = 32 + 4 = 36 \]
   \[ \left( x + \frac{2}{3} \right)^2 = \frac{36}{9} = 4 \]

6. Set the given fraction equal to \( x \).
   Then
   \[ x = \frac{3}{2 + x} \]
   \[ x^2 + 2x - 3 = 0 \]
   \[ (x - 1)(x + 3) = 0 \]
   Clearly, \( x > 0 \), so \( x = 1 \)