2007-08 Event 3A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. List all quadrants (using the symbols I, II, III, IV) that contain at least one point satisfying both \( y > 2x \) and \( y > 4 - x \).

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2. Ann and Sue bought identical boxes of stationery (which contain some paper and some envelopes). Ann used her stationery to write 1-sheet letters and Sue used hers to write 3-sheet letters. Ann used all of her envelopes and had 50 sheets of paper left, while Sue used all of her sheets of paper and had 50 envelopes left. How many sheets of paper were in a box of stationery?

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3. A teenage boy wrote his own age after his father’s age, creating a four-digit number. From this number he subtracted the absolute value of the difference of their ages, yielding 4289. Find the boy’s age.

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4. Four positive integers are given. Suppose you select three of the integers, find their arithmetic mean, and add this mean to the fourth integer, calling your final result \( n \). If the possible values of \( n \) are 29, 23, 21, and 17, what were the original four integers?

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Name ___________________________ Team ___________________________
Minnesota State High School Mathematics League
Individual Event

2007-08 Event 3B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. In parallelogram $ABCD$ (Figure 1) with height $h = 8$, $AB = 10$ and $BC = 18$, find the length of the shorter diagonal ($AC$).

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2. Let $KLMN$ (Figure 2) be a trapezoid with longer base $KL$. $FG$ is drawn parallel to both bases, with $F$ on $KN$ and $G$ on $LM$. Given that $KN = 35$, $GL = 12$, $FN = x$, and $FK = 2x - 7$, find $GM$ (without involving $x$ in your answer).

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3. Given a rhombus $PQRS$ (Figure 3) with $m\angle PQR = 60^\circ$ and altitude $PT = 27$, find the length of the longer diagonal ($QS$).

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4. Given the three points $A(-5, 5), B(15, 20), C(24, 11)$ in the coordinate plane, we want to add a fourth point $D$ in Quadrant IV to make a trapezoid where $m\angle ADC = 90^\circ$. Give an ordered pair of coordinates for $D$.

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Name ____________________________ Team ____________________________
The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. If \( f(x) = \tan^{-1} x \), and \( f(a) = b \), express \( f(-a) \) in terms of \( b \).

2. Find the value of \( n \) for which \((1+i)^n = 16\).

3. A friend recently showed me a faulty surveyor’s diagram of his property in northern Minnesota, on which he has placed a permanent trailer (Figure 3). The only measurements he could trust were that the two eastern corners of the property (marked \( A \) and \( B \)) are 184.79 feet apart and corner \( A \) is 200 feet from the trailer hitch (\( H \)). I used an orientation compass to determine that \( \overline{AH} \) and \( \overline{AB} \) are oriented 308° and 18° clockwise of north respectively. Find the distance from corner \( B \) to the trailer hitch \( H \).

4. Let \( \overline{AB} \) be the diameter of a semicircle. Place point \( C \) on the semicircle’s arc so that the area of \( \triangle ABC \) equals \( \frac{1}{2} \) the area of the semicircle. The measure of \( \angle ABC \), the smallest angle in the triangle, can be written in many ways – but one of the simplest is the form \( q_1 \cdot \sin^{-1}(q_2 \cdot \pi) \), where \( q_1 \) and \( q_2 \) are rational numbers. Find \( q_1 \) and \( q_2 \).

\[ q_1 = \quad q_2 = \]

Name

Team
The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. Find the value of $x$ that satisfies the equation $\log_x 25 - \log_x 4 = \log_x \sqrt{x}$.

2. If $\frac{4^x}{8^y} = \frac{8^{x+1}}{16^{y-1}}$, compute the value of $x - y$.

3. Find all solutions to the equation $\log x^2 = 2 \log (3x + 7)$.

4. If $a = b^b$, $b = c^c$, and $\log_c (\log_x a) = \pi$, compute the exact value of $c$.

Name ___________________________ Team ___________________________
Minnesota State High School Mathematics League
Team Event

2007-08 Meet 3

Each question is worth 4 points. Team members may cooperate in any way, but at the end of 20 minutes, submit only one set of answers. Place your answer to each question on the line provided.

1. If \( \log_a b = \log_{8a} (16b) = \log_{8b} 16 \), compute the value of \( a \), in simplest radical form.

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2. In counting \( n \) colored balls, some red and some black, it was found that 49 of the first 50 balls counted were red. After that, 7 out of every 8 counted were red. If, in all, at least 90% of the balls were red, what is the maximum possible value for \( n \)?

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3. In \( \triangle APC \) (Figure 3), \( m \angle EPF = 20^\circ \), \( m \angle AFE = 30^\circ \), and \( \triangle PBF \sim \triangle ABE \). If \( AC = 7 \) and \( AP = 9 \), find the measure of acute \( m \angle ACP \).

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4. When the system \( 3x - 4y < 6 \) and \( 6x + 4y = 15 \) is solved, all solutions \((x, y)\) have the form \( \left( \frac{7 - r}{3}, \frac{1 + s}{4} \right) \).

   Find the ratio \( r : s \).

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5. In trapezoid \( ABCD \) (Figure 5), \( AB \parallel CD \).

   If \( \text{Area}(\triangle ABE) = \log_7 e \), \( \text{Area}(\triangle CDE) = \log_7 a \), and \( \text{Area}(\triangle ABC) = 5 \), compute the area of \( ABCD \).

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6. In Figure 6, \( X \) and \( W \) are the midpoints of \( TU \) and \( UV \) respectively. If \( XY = 4 \), \( WU = 6 \), and \( UX = 7 \), find \( m \angle XUW \) to the nearest degree.

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