1998-99 Event 2B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. In the figure, the right \( \triangle AOB \) has \( OA = 2 \) and \( AB = 1 \). Also, \( \triangle OAB \sim \triangle OBC \sim \triangle OAD \). Find the length of \( OD \).

Repeated use of Pythagorean Theorem, together with properties of similar triangles, yields the numbers in the figure.

2. A square is inscribed in a 3-4-5 right-angled triangle as shown. What fraction of the triangle does it occupy? From the similarity of triangles 1 and 2,

\[
\frac{3-x}{x} = \frac{2}{4-x}, \quad 12-7x+x^2 = x^2
\]

\[
(2-x)(4-x) = x^2
\]

\[
x = \frac{12}{7}
\]

3. In the figure, right triangle \( OAB \) has \( OA = 2 \), \( AB = 1 \), and \( C \) is the foot of the altitude from \( A \). Find the area of \( \triangle ABC \).

\[
x^2+y^2 = 1 \quad \text{and} \quad \frac{x}{y} = \frac{1}{2}, \text{ so } y = 2x,
\]

\[
x^2+(2x)^2 = 1 \Rightarrow x = \frac{1}{\sqrt{5}}, \quad y = \frac{2}{\sqrt{5}}, \quad \text{Area } \triangle ABC = \frac{1}{2} \cdot \frac{1}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}} = \frac{1}{5}.
\]

4. In the space to the right, draw a geometric figure to show that for all pairs of real numbers \( (x,y) \), \( \sqrt{x^2+y^2} + \sqrt{(x-4)^2 + (y-3)^2} \geq 5 \).

From the figure,

\[
|OP| + |PA| \geq OA.
\]

Draw and label all points. **Lengths on the diagram.** 1 pt

Explain its meaning. 1 pt