Answers

Minnesota State High School
Mathematics League
Individual Event

2001-02 Event 1B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. Figure 1 shows a regular pentagon with center O. Find the measure of \( \angle OAB \).

2. Figure 2 shows the right \( \triangle AFG \) inscribed in a rectangle. Express the measure of \( \angle CFG \) in terms of \( \alpha \) and \( \beta \) where \( \alpha = \angle FAB \) and \( \beta = \angle GAF \).

3. From Figure 2, express the measure of \( \angle AHG \) in terms of \( \alpha \) and \( \beta \).

4. Refer again to the regular pentagon shown in Figure 1. Extend sides \( AB \) and \( BC \) to \( B' \) and \( C' \) respectively so that \( BB' = CC' \) and \( \angle BB'C = 90^\circ \). Denoting the center of the pentagon by \( O \), find the measure of \( \angle AOB' \) to the nearest degree.

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1. \( m(\angle OAB) = \frac{1}{2} \left[ \frac{3}{5} (180^\circ) \right] = 54^\circ \)

2. \( \angle CFG = \angle FGH \) (alt. interior angles)
\( \angle FGH = \alpha \) (sides mutually 1)

3. \( \beta + \angle AGH + \alpha + 90 = 180 \)
\( \angle AGH = 90 - (\alpha + \beta) \)
\( \beta + \angle AGH + \angle AHG = 180 \)
\( \angle AHG = 180 - \beta - [90 - (\alpha + \beta)] \)
\( = 90 + \alpha \)

4. Designate by \( \Theta \)
\( \angle LOA' = \angle LOB = \angle LOBC \)

Also from (1), \( \angle OCB = \angle OBA = 90 - \omega \)

Thus \( \angle AOB' = 180 - [\omega + \angle OBA] = 180 - [\omega + (90 - \omega)] \)

It will follow that \( \angle AOB' = 90 \) if we show that \( \omega = \Theta \).

\( 90 - \omega = \angle OCB = \angle OCB \)
\( 90 - \omega = \angle OCB' = \angle OCB \)

\( \Rightarrow \text{Alternately (and easier for 3) \angle AHG is an exterior angle of } \triangle HFG \text{ that is, therefore, the sum of the two remote interior } \angle \text{s.} \)