Solutions

Minnesota State High School Mathematics League
Individual Event

2002-03 Event 2D

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

54. 1. What is the area in the first quadrant cut off by the graph of
\[ \frac{x}{12} + \frac{y}{9} = 1 \]

\[ (7,4) \] 2. A line cuts the x-axis at \( \frac{7}{3} \), the y-axis at -2. Give the coordinates of the lattice point (a point having integers as coordinates) on this line, lying in the first quadrant, that has the smallest y coordinate.

3. The x and y intercepts of a line are both negative, with the x-intercept being three times the y-intercept. The distance of the line from the origin is 6. Write the equation of the line in the standard form \( Ax + By + C = 0 \) with \( A, B \) and \( C \) in exact form.

\[ x + 3y + 6\sqrt{10} = 0 \]

\[ \left( \frac{a}{2}, -\frac{b}{2} \right) \] The graph of the line \( y = 2x - 1 \) is the perpendicular bisector of the line segment AB. The coordinates of A are \( \left( -\frac{3}{2}, -\frac{11}{2} \right) \). What are the coordinates of B?

1. \[ A = \frac{1}{2} (12)(9) = 54 \]

2. \[ \frac{x}{9} + \frac{y}{-2} = 1 \]
\[ y = \frac{6}{7}x - 2 \]
When \( x = 7, y = 4 \)

3. \[ b \]

4. \[ \vec{AM} = \left( t + \frac{3}{2} \right) i + \left( -2t - 1 + \frac{11}{2} \right) j \]

\[ \text{slope } \vec{AM} = \frac{-2t + \frac{9}{2}}{t + \frac{3}{2}} = \frac{1}{2} \]

Solution gives \( t = \frac{3}{2}, s \)

\[ \vec{AM} = 3i + \frac{3}{2}j \]

\[ \overrightarrow{OB} = \overrightarrow{OM} + \overrightarrow{MB} = \overrightarrow{OM} + \overrightarrow{AM} = \frac{a}{2} i - \frac{5}{2} j \]