2002-03 Meet 2

Each question is worth 4 points. Team members may cooperate in any way, but at the end of twenty minutes, one set of answers is to be submitted. Put answers on the lines provided.

1. Two ferry boats ply back and forth between an island and the mainland with constant but different speeds. They leave opposite shores at the same time, meet for the first time \( \frac{3}{4} \) of a mile from one shore, continue on their way, take 20 minutes to unload and load at their respective destinations, and meet for the second time \( \frac{1}{2} \) mile from the opposite shore. What is the distance from the mainland to the island?

2. [NEAML] Find all three-digit numbers of the form ABA, A and B being distinct digits (and of course \( A \neq 0 \)), such that ABA is divisible by both 3 and 11.

3. Locate a point \( P \) that is equidistant from \( A\left(-\frac{2}{3}, \frac{9}{2}\right) \), \( B\left(\frac{3}{2}, 1\right) \), and the \( x \)-axis.

4. [NEAML] If \( \sin 2x = 2(\sin x + \cos x) \), determine all possible values of \( (\sin x)(\cos x) \).

5. A moving point is one third as far from the origin as it is from the graph of \( x + y = 4 \). Express the equation of its path as a polynomial in \( x \) and \( y \).

6. The graph of \( y = \sin x + \cos x \) rises to its maximum value once for \( 0 \leq x \leq 2\pi \). How many times does the graph of \( y = \sin \frac{1}{x} + \cos \frac{1}{x} \) rise to its maximum for \( 0.0001 \leq x \leq 2 \)?