Minnesota State High School Mathematics League
Individual Event

2005-06 Event 2A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

This is a no calculator event!

1. What value of \( x \) satisfies \( 16 - x = x - 2 \)?

2. Express \( x \) as a single quotient of terms involving \( a \) and \( b \), given that \( a^2 x - b = b^2 x - a \).

3. A yardstick casts a shadow of 27 inches at the same time that a tree in our back yard casts a shadow of 40 feet. How high (give feet and inches) is the tree?

4. What value(s) of \( x \) make \( f(x) = |x + 2| + |x - 1| \) as small as possible?
2005-06 Event 2B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

All questions refer to the trapezoid RSTU at the right. It has bases $a = RS$ and $b = UT$; $VW$ is parallel to $RS$.

1. If $VW$ is placed so that trapezoids $RSWV$ and $VWTU$ are similar, express the length of $VW$ in terms of $a$ and $b$.

2. If $VW$ is placed so that it bisects $RU$, express the length of $VW$ in terms of $a$ and $b$.

3. Suppose that the figure is drawn with $RU \perp RS$. Suppose further that the diagonals are perpendicular; i.e. $US \perp RT$. Express the height of trapezoid $RSTU$ in terms of $a$ and $b$.

4. If $VW$ is placed so that it passes through the intersection $K$ of diagonals $US$ and $RT$, express $VW$ in terms of $a$ and $b$.

Name __________________________ Team _________________________
2005-06 Event 2C
The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. If $\theta$ is a second quadrant angle for which $\sin \theta = \frac{3}{5}$, what is $\sin 2\theta$?

2. If $\theta$ is a second quadrant angle for which $\sin \theta = \frac{\sqrt{5}}{3}$, then $\cos \frac{\theta}{2}$ can be expressed in the form $\frac{\sqrt{k}}{k}$ where $k$ is an integer. Find $k$.

3. In the right $\triangle ABC$ with leg $AB = 2$ and hypotenuse $AC = 5$, the bisector of $\angle A$ meets $BC$ at $D$. Express the length of $BD$ in the form $r\sqrt{n}$ where $r$ is rational and $n$ is an integer.

4. In $\triangle ABC$ with $\angle A = 60^\circ$, $b = AC = 6$ and $c = AB = 4$. The bisector of $\angle A$ meets $BC$ at $D$. Express the length of $AD$ in the form $r\sqrt{n}$ where $r$ is rational and $n$ is an integer.
Minnesota State High School Mathematics League
Individual Event

2005-06 Event 2D

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

All questions in this event refer to the figure at the right that shows:
- a circle $\Gamma$ of radius 2 centered at the origin
- a circle $\Omega$ of radius 3 centered at $C(7,0)$
- a line tangent to $\Omega$ and $\Gamma$ at $S$ and $T$
- a line tangent to $\Omega$ and $\Gamma$ at $M$ and $N$
- a circle tangent to $\Omega$ and $\Gamma$ at $S$ and $P$.

1. Write the equation of the circle centered on the $x$-axis that contains neither $\Omega$ nor $\Gamma$, but is tangent to them both.

2. Where does the line $ST$ cross the $x$-axis?

3. What is the slope of the line $MN$?

4. Where does the line $SP$ cross the $x$-axis?

Name_________________________________________ Team_________________________________________
Minnesota State High School Mathematics League
Team Event

2005-06 Meet 2
Each question is worth 4 points. Team members may cooperate in any way, but at the end of twenty minutes, one set of answers is to be submitted. Put answers on the lines provided.

1. In \( \triangle ABC \), \( a = BC \) and \( b = AC \). The bisector of \( \angle C \) meets \( AB \) at \( D \). Express the length of \( CD \) in terms of \( a, b, \) and \( \theta \) where \( \theta = \frac{1}{2} \angle C \).

2. Figure 2 shows a circle \( \Gamma \) of radius 2 centered at the origin and a circle \( \Omega \) of radius 3 centered at \( C(7,0) \).
   
   (a) From \( O \), tangents drawn to \( \Omega \) intersect \( \Gamma \) at \( A \) and \( B \). Find the length of \( AB \).
   
   (b) From \( C \), tangents drawn to \( \Gamma \) intersect \( \Omega \) at \( E \) and \( F \). Find the length of \( EF \).

3. Compute the area of the circle that passes through all the intersections of
   \[ 4x^2 + 11y^2 = 29 \] and \[ x^2 - 6y^2 = 6. \]

4. For a third degree polynomial \( p(x) \), \( p(n) = 2^n \) for \( n = 0, 1, 2, 3 \). Find \( p(4) \).

5. With \( O \) as the origin, a right \( \angle AOB \) is inscribed in the parabola \( y^2 = 4x \) so that the slope of \( OA \) is \( \frac{1}{2} \). Where does \( AB \) cross the x-axis?

6. In the right \( \triangle ABC \), \( DE \) is parallel to \( AC \) and partitions the triangle into regions having equal areas (Figure 6). Let \( a = CB, b = CA, \) and \( x = ED \). Given that \( a + b = 70 \) and \( a - b = x \), express \( b \) in the form \( k + m\sqrt{n} \) where \( k, m, \) and \( n \) are integers.

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Figure 2

Team ____________________________

Figure 6