Minnesota State High School
Mathematics League
Individual Event

2006-07 Tournament Event A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 15 minutes for this event.

1. Express \[ \frac{x^4 - xy^3 + x^3 y - y^4}{x^2 - y^2} \] as a polynomial in \( x \) and \( y \).

2. Jane is half as old as Terry will be in six years when the sum of their ages will be 75. How old is Jane now?

3. Solve the system

   \[
   \begin{align*}
   3y - 5z &= 4 \\
   x + 6y + 5z &= 3 \\
   3y - z &= 2
   \end{align*}
   \]

4. Find the largest value of \( s \) and the smallest value of \( t \) such that for all \( x \) satisfying \(-6 < -2x < 8\) and for all \( y \) satisfying \( 1 < |y - 4| < 3\),

   (a) \( s = \quad \quad t = \quad \quad (a) \quad \quad s < xy < t \)

   (b) \( s = \quad \quad t = \quad \quad (b) \quad \quad s < \frac{x}{y} < t \)

STICK YOUR "NAME/TEAM" LABEL ON THE BACK OF THE EXAM
2006-07 Tournament Event B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 15 minutes for this event.

1. Figure 1 shows a square $ABCD$ with sides of length 4. Concentric circles, one inscribed in the square, the other circumscribed about it, bound a washer shaped region. What is the area of this region?

2. The difference between the largest and smallest angles of a parallelogram is $66^\circ$. What is the measure of the smallest angle?

3. Figure 3 shows a square $ABCD$ with sides of length 4, and four inscribed semicircles centered at $E$, $F$, $G$, and $H$. These semicircles intersect at $K$ and overlap to form four (shaded) leaves. What is the total area of the four shaded leaves?

4. Figure 4 shows four points connected by six lines, three short ones each having length 1, and three others each having the same length $x > 1$. Find $x$, accurate to three places to the right of the decimal.

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The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 15 minutes for this event.

1. Let $\alpha$ and $\beta$ be two angles, $0 < \alpha < \beta < 2\pi$, such that $\sin \alpha = \sin \beta = -\frac{1}{2}$. What is the radian measure of $\beta - \alpha$?

2. Figure 2 shows an equilateral triangle inscribed in a right isosceles triangle having legs of length 4. Give the length of the sides of the equilateral triangle in exact, rationalized form.

3. A cooler contains 4 bottles of carbonated mineral water and 6 bottles of non-carbonated spring water. Not realizing there is a choice, Alicia and Beth each plunge their hands into the ice and grab a bottle. What is the probability that both girls get the same kind of water?

4. A complex number $a + bi$ lies on the unit circle if $a^2 + b^2 = 1$. Express each of the two roots of $x^2 = 4 + 3i$ in the form $r(a + bi)$ where the real numbers $r$, $a$, and $b$ are expressed in exact form and $a + bi$ lies on the unit circle.

Figure 2

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2006-07 Tournament Event D

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is
worth 2 points. Place your answer to each question on the line provided. You have 15 minutes for this
event.

THIS IS A NO CALCULATOR EVENT

1. Express $4^{3^3} - (4^3)^2$ as a single integer.

2. Point A is in the first quadrant, located so that with O designating the origin,
   $OA = 2$ and $OA$ makes an angle of $30^\circ$ with the positive $x$-axis. Write an equation of
   the line through $A$, perpendicular to $OA$.

3. The quadratic equation $x^2 + bx + c = 0$ has roots $1 + m + \sqrt{1 + m^2}$ and $\frac{2m}{1 + m + \sqrt{1 + m^2}}$.
   Express $b$ as a linear expression in $m$.

4. Using common logarithms (that is, logarithms to the base 10), $\log 5 = 0.69897$. Find
   $\log \sqrt{8}$ rounded correct to three places to the right of the decimal.

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Each question is worth 4 points. Team members may cooperate in any way, but at the end of thirty minutes, one set of answers is to be submitted. Put answers on the lines provided.

1. The parallelogram \(ABCD\) with \(EF \parallel AB\) and \(\angle BAD = 60^\circ\) (Figure 1) is often used to create an optical illusion. The viewer is asked which is longer, \(BE\) or \(EC\). Suppose \(AB = 1\) and \(BC = 2\). How long should \(AE\) be so that \(BE = EC\)?

2. The polynomial \(p(x)\) has non-negative coefficients; \(p(1) = 12\) and \(p(20) = 18,081\). Find \(p(x)\).

3. Write equations for all lines passing through \((1, -1)\) having an x-intercept that is twice the slope of the line.

4. The mean of the set of real numbers \(\{x_1, x_2, \ldots, x_n\}\) is \(k\). The mean of the set \(\{x_1, x_2, \ldots, x_n, 17\}\) is 8. What is \(k\)?

5. \(P\) is a point in the interior of \(\triangle ABC\) that has \(AB = 13\), \(BC = 7\), and \(CA = 8\). The distance from \(P\) to the side \(AB\) is \(\frac{\sqrt{3}}{4}\); its distance to the side \(BC\) is \(\frac{5\sqrt{3}}{4}\). Find the distance from \(P\) to the side \(AC\).

6. Here is a system of three equations in the three unknowns \(x, y,\) and \(z\).

\[
\begin{align*}
x + ay + a^2z &= a^3 \\
x + by + b^2z &= b^3 \\
x + cy + c^2z &= c^3
\end{align*}
\]

The unknown \(x\) can be written as a single term involving only of \(a, b,\) and \(c\). Find this term.