1. Write a linear inequality that describes the graph shown in Figure 1.

2. Solve for $x$: $x(x + 1) + 2 = x(x - 3) - 4$.

   $x =$ __________________________

3. When looking at two particular 3-digit positive integers, I discovered that if I placed the numbers side-by-side, and then placed a decimal point between them, the result would be the mean of the original two numbers. What two integers was I looking at? (Note: 3-digit numbers must begin with a non-zero digit.)

   __________  __________

4. If $c = a - b(a - b(a - b(a - ...))))$, find an expression for $b$ that uses the variables $a$ and $c$ only once each.

   $b =$ __________________________

Name ___________________________  Team ___________________________
1. In Figure 1, the diagonals of square $ABCD$ meet at $P$. If $AP = 3$, compute the exact length of $CD$.

$CD =$

2. Square $EFGH$ is inscribed inside square $WXYZ$, as shown in Figure 2. If $m \angle WEF = 30^\circ$ and $EF = 4$, compute the length of diagonal $XZ$.

$XZ =$

3. In triangle $KLM$ (Figure 3), $m \angle K = 60^\circ$ and $m \angle L = 75^\circ$. If $KL = 4$, compute the perimeter of $\triangle KLM$.

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4. In triangle $ABC$ (Figure 4), altitudes $\overline{AP}$, $\overline{BQ}$, and $\overline{CR}$ intersect at $H$. Given $BP = 4$, $HP = 3$, and $HQ = 2$, compute the exact length of $AC$.

$AC =$

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Minnesota State High School Mathematics League
Individual Event

2009-10 Event 2C

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

NO CALCULATORS are allowed on this event.

1. If \( \tan \theta = \frac{\tan 23^\circ - \tan 37^\circ}{1 + \tan 23^\circ \tan 37^\circ} \), find one possible value for \( \theta \), in degrees.

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2. Simplify \( \sin (x + 3\pi) \), writing it as an expression involving only a single trigonometric function of \( x \).

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3. Given that \( \alpha \) is the smallest acute angle of a 5-12-13 right triangle, evaluate \( \frac{\cos 2\alpha}{\sin 2\alpha} \), expressing your answer as the quotient of two relatively prime integers.

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4. If \( x = \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ \), then \( x \) can be written as a single rational number. Do so.

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Name ___________________________ Team ___________________________
All questions in this event refer to the circle \( C \), described by the equation:
\[
(x - 4)^2 + (y - 3)^2 = 25.
\]

1. Indicate the center and radius of circle \( C \).
   
   \[
   \text{center} = (\quad,\quad) \quad \text{radius} = \quad
   \]

2. Circle \( C \) passes through the origin. Where else does it intersect the \( x \)-axis?

   
   

3. A circle concentric with \( C \) passes through \( A (5,5) \). Let \( \overline{BD} \) be a diameter of this circle that is parallel to the \( x \)-axis. What will be the area of \( \triangle ABD \) ?

   
   

4. A circle concentric with \( C \) is tangent to the line \( y = \frac{1}{2}x + 3 \) at a point \( T \).
   Find the coordinates of \( T \).

   \[
   T = \quad
   \]

Name ________________________________  Team ________________________________
Each question is worth 4 points. Team members may cooperate in any way, but at the end of 20 minutes, submit only one set of answers. Place your answer to each question on the line provided.

1. In isosceles triangle $\triangle ABC$ (Figure 1), altitudes $AP$ and $BQ$ have length 4, and altitude $CR$ has length 5. Compute the exact perimeter of $\triangle ABC$.

2. Compute the exact value of the sum:

$$\cos\left(\frac{\pi}{12}\right) + \cos\left(\frac{2\pi}{12}\right) + \cos\left(\frac{3\pi}{12}\right) + \ldots + \cos\left(\frac{2008\pi}{12}\right) + \cos\left(\frac{2009\pi}{12}\right)$$

3. I’m thinking of two numbers. One number is three times the other, and their sum is 8 more than twice the lesser number. What is the least possible number I could be thinking of?

4. In a certain $30^\circ - 60^\circ - 90^\circ$ triangle, the median and altitude to the hypotenuse have lengths that sum to 1. The perimeter of this triangle can be expressed in the form $3a - b\sqrt{3}$. Find the ordered pair $(a, b)$.

$(a, b) =$

5. The line $y = \frac{5}{12}x$ is tangent to a member of the family of circles described by $(x - h)^2 + (y - 3)^2 = h^2$, where $h > 0$. Compute $h$.

$h =$

6. In quadrilateral $ABCD$ (Figure 6), $AB = 8$, $CD = 14$, and $2 \cdot m\angle A = 7 \cdot m\angle B = 7 \cdot m\angle C = 14 \cdot m\angle D$ describes a relationship among the interior angles. Compute the exact area of the quadrilateral.

Team ____________________________